Section 7.3

Check Your Understanding, page 456:

1. **Step 1: State the distribution and values of interest.** X = length of pregnancy follows a $N(266, 16)$ distribution and we want to find $P(X > 270)$. **Step 2: Perform calculations. Show your work.** The standardized score for the boundary value is $z = \frac{270-266}{16} = 0.25$. The desired probability is $P(Z > 0.25) = 1 - 0.5987 = 0.4013$. *Using technology*: normalcdf(lower: 270, upper: 1000, μ: 266, σ: 16) = 0.4013. **Step 3: Answer the question.** There is a 0.4013 probability of selecting a woman whose pregnancy lasts for more than 270 days.
2. The mean of the sampling distribution of $\bar{x}$ is equal to the mean of the distribution of X so $\mu_{\bar{x}} = \mu = 266$ days.
3. The sample of size 6 is less than 10% of all pregnant women. Therefore, the standard deviation of the sampling distribution of $\bar{x}$ is $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{16}{\sqrt{6}} = 6.532$ days.
4. **Step 1: State the distribution and values of interest.** $\bar{x}$ = mean length of pregnancy follows a $N(266, 6.532)$ distribution and we want to find $P(\bar{x} > 270)$. **Step 2: Perform calculations. Show your work.** The standardized score for the boundary value is $z = \frac{270-266}{6.532} = 0.61$. The desired probability is $P(Z > 0.61) = 1 - 0.7291 = 0.2709$. *Using technology*: normalcdf(lower: 270, upper: 1000, μ: 266, σ: 6.532) = 0.2701. **Step 3: Answer the question.** There is a 0.2701 probability of selecting a sample of 6 women whose mean pregnancy length exceeds 270 days.

Exercises, page 461:

7.49 The mean of the sampling distribution of $\bar{x}$ is $\mu_{\bar{x}} = \mu = 225$ seconds. Because the sample size (10) is less than 10% of the population of songs on David's iPod, the standard deviation is $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{60}{\sqrt{10}} = 18.974$ seconds. These results do not depend on the shape of the distribution of the individual play times.

7.50 The mean of the sampling distribution of $\bar{x}$ is $\mu_{\bar{x}} = \mu = 40.125$ mm. Assuming the sample size (4) is less than 10% of all axles produced this hour, the standard deviation is $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.002}{\sqrt{4}} = 0.001$ mm. These results do not depend on the shape of the distribution of the individual axle diameters.

7.51 If we want $\sigma_{\bar{x}} = 30$, then we need to solve the following equation for n:

$30 = \frac{60}{\sqrt{n}} \rightarrow \sqrt{n} = \frac{60}{30} = 2 \rightarrow n = 4$. So we need a sample of size 4.

7.52 If we want $\sigma_{\bar{x}} = 0.0005$, then we need to solve the following equation for n:

$0.0005 = \frac{0.002}{\sqrt{n}} \rightarrow \sqrt{n} = \frac{0.002}{0.0005} = 4 \rightarrow n = 16$. So we need a sample of size 16.

7.53 (a) The sampling distribution of $\bar{x}$ is Normal with $\mu_{\bar{x}} = \mu = 188$ mg/dl. Because the sample size (100) is less than 10% of all men aged 20 to 34, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{41}{\sqrt{100}} = 4.1$mg/dl.

(b) **Step 1: State the distribution and values of interest.** $\bar{x}$ = mean cholesterol level follows a *N*(188, 4.1) distribution and we want to find $P(185 \le \bar{x} \le 191)$. **Step 2: Perform calculations. Show your work.** The standardized scores for the boundary values are $z = \frac{185-188}{4.1} = -0.73$ and $z = \frac{191-188}{4.1} = 0.73$. The desired probability is $P(-0.73 \le Z \le 0.73) = 0.7673 - 0.2327 = 0.5346$. *Using technology*: normalcdf(lower: 185, upper: 191, μ: 188, σ: 4.1) = 0.5357. **Step 3: Answer the question.** There is a 0.5357 probability that $\bar{x}$ estimates μ within ± 3 mg/dl.

(c) In this case $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{41}{\sqrt{1000}} = 1.30$mg/dl. So $\bar{x}$ = mean cholesterol level follows a *N*(188, 1.30) distribution and we want to find $P(185 \le \bar{x} \le 191)$. **Step 2: Perform calculations. Show your work.** The standardized scores for the boundary values are $z = \frac{185-188}{1.30} = -2.31$and $z = \frac{191-188}{1.30} = 2.31$. The desired probability is $P(-2.31 \le Z \le 2.31) = 0.9896 - 0.0104 = 0.9792$. *Using technology*: normalcdf(lower: 185, upper: 191, μ: 188, σ: 1.30) = 0.9790. **Step 3: Answer the question.** There is a 0.9790 probability that $\bar{x}$ estimates μ within ± 3 mg/dl. The larger sample is better because it is more likely to produce a sample mean within 3 mg/dl of the population mean.

7.54 (a) The sampling distribution of $\bar{x}$ is Normal with $\mu_{\bar{x}} = \mu = 48$ months. Because the sample size (8) is less than 10% of all batteries, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{8.2}{\sqrt{8}} = 2.899$ months.

(b) **Step 1: State the distribution and values of interest.** $\bar{x}$ = mean battery lifetime follows a *N*(48, 2.899) distribution and we want to find $P(\bar{x} \le 42.2)$. **Step 2: Perform calculations. Show your work.** The standardized score for the boundary value is $z = \frac{42.2-48}{2.899} = -2.00$. The desired probability is $P(Z \le -2.00) = 0.0228$. *Using technology*: normalcdf(lower: –1000, upper: 42.2, μ: 48, σ: 2.899) = 0.0227. **Step 3: Answer the question.** There is a 0.0227 probability that the sample mean lifetime is 42.2 months or less if the lifetime distribution is unchanged. Because this probability is very small, there is convincing evidence that the lifetime distribution has changed. It is not plausible to get a sample mean this small by chance alone.

7.55 (a) **Step 1: State the distribution and values of interest.** Let X = amount of cola in a randomly selected bottle. X follows the $N(298, 3)$ distribution and we want to find $P(X < 295)$. **Step 2: Perform calculations. Show your work.** The standardized score for the boundary value is $z = \frac{295-298}{3} = -1$. The desired probability is $P(Z < -1) = 0.1587$. *Using technology*: normalcdf(lower: –1000, upper: 295, μ : 298, σ : 3) = 0.1587. **Step 3: Answer the question.** There is a 0.1587 probability that a randomly selected bottle contains less than 295 ml.
(b) **Step 1: State the distribution and values of interest.** Because X has a Normal distribution, the sampling distribution of $\bar{x}$ has a Normal distribution. $\mu_{\bar{x}} = \mu = 298$ ml. Because 6 is less than 10% of all bottles produced, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3}{\sqrt{6}} = 1.2247$ ml. We want to find $P(\bar{x} < 295)$ using the $N(298, 1.2247)$ distribution. **Step 2: Perform calculations. Show your work.** The standardized score for the boundary value is $z = \frac{295-298}{1.2247} = -2.45$. The desired probability is $P(Z < -2.45) = 0.0071$. *Using technology*: normalcdf(lower: –1000, upper: 295, μ : 298, σ : 1.2247) = 0.0072. **Step 3: Answer the question.** There is a 0.0072 probability that the mean contents of six randomly selected bottles is less than 295 ml.

7.56 (a) **Step 1: State the distribution and values of interest.** Let X = amount of cereal in a randomly selected box. X follows the $N(9.70, 0.03)$ distribution and we want to find $P(X < 9.65)$. **Step 2: Perform calculations. Show your work.** The standardized score for the boundary value is $z = \frac{9.65-9.70}{0.03} = -1.67$. The desired probability is $P(Z < -1.67) = 0.0475$. *Using technology*: normalcdf(lower: –1000, upper: 9.65, μ : 9.70, σ :0.03) = 0.0478. **Step 3: Answer the question.** There is a 0.0478 probability that a randomly selected box contains less than 9.65 ounces.
(b) **Step 1: State the distribution and values of interest.** Because X has a Normal distribution, the sampling distribution of $\bar{x}$ has a Normal distribution. $\mu_{\bar{x}} = \mu = 9.70$ ounces. Because 5 is less than 10% of all boxes produced, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.03}{\sqrt{5}} = 0.0134$ ounces. We want to find $P(\bar{x} < 9.65)$ using the $N(9.70, 0.0134)$ distribution. **Step 2: Perform calculations. Show your work.** The standardized score for the boundary value is $z = \frac{9.65-9.70}{0.0134} = -3.73$. The desired probability is $P(Z < -3.73) \approx 0$. *Using technology*: normalcdf(lower: –1000, upper: 9.65, μ : 9.70, σ : 0.0134) = 0.0001. **Step 3: Answer the question.** There is a 0.0001 probability that the mean contents of 5 randomly selected boxes is less than 9.65 ounces.

7.57 No. The histogram of the sample values will look like the population distribution, whatever it might happen to be. The central limit theorem says that the histogram of the sampling distribution of the *sample mean* will look more and more Normal as the sample size increases.

7.58 Although this is a correct statement about the standard deviation of the sampling distribution of $\bar{x}$, this is not what the Central Limit Theorem says. The Central Limit Theorem addresses the *shape* of the sampling distribution, not the spread.

7.59 (a) Because the distribution of the play times of the population of songs is heavily skewed to the right, a sample size of 10 is not large enough to assume that the distribution of $\bar{x}$ is approximately Normal.
(b) Because $n = 36 \geq 30$, the sample size is large enough for the Central Limit Theorem to apply. **Step 1: State the distribution and values of interest.** $\mu_{\bar{x}} = \mu = 225$ seconds. Because 36 is less than 10% of all songs on David's iPod, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{60}{\sqrt{36}} = 10$ seconds. We want to find $P(\bar{x} > 240)$ using the $N(225, 10)$ distribution. **Step 2: Perform calculations. Show your work.** The standardized score for the boundary value is $z = \frac{240 - 225}{10} = 1.50$. The desired probability is $P(Z > 1.50) = 1 - 0.9332 = 0.0668$. *Using technology*: normalcdf(lower: 240, upper: 1000, μ: 225, σ: 10) = 0.0668. **Step 3: Answer the question.** There is a 0.0668 probability that the mean play time is more than 240 seconds.

7.60 (a) If $\bar{x}$ is the mean number of strikes per square kilometer, then $\mu_{\bar{x}} = \mu = 6$ strikes/km^2 and because 10 is less than 10% of all one-square-kilometer plots of land, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2.4}{\sqrt{10}} = 0.7589$ strikes/km^2.
(b) Because we do not know the shape of the distribution of the number of lightning strikes, a sample of size 10 is not large enough to assume that the distribution of $\bar{x}$ is approximately Normal.
(c) With a large sample size ($n = 50 \geq 30$), the Central Limit Theorem assures us that the sampling distribution of $\bar{x}$ is approximately Normal. **Step 1: State the distribution and values of interest.** $\mu_{\bar{x}} = \mu = 6$ strikes/km^2. Because 50 is less than 10% of all one-square-kilometer plots, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2.4}{\sqrt{50}} = 0.3394$ strikes/km^2. We want to find $P(\bar{x} < 5)$ using the $N(6, 0.3394)$ distribution. **Step 2: Perform calculations. Show your work.** The standardized score for the boundary value is $z = \frac{5 - 6}{0.3394} = -2.95$. The desired probability is $P(Z < -2.95) = 0.0016$. *Using technology*: normalcdf(lower: –1000, upper: 5, μ: 6, σ: 0.3394) = 0.0016. **Step 3: Answer the question.** There is a 0.0016 probability that the mean number of lightning strikes per kilometer is less than 5.

7.61 (a) The probability that a single passenger weighs more than 200 pounds cannot be calculated, because we do not know the shape of the distribution of passenger weights.
(b) **Step 1: State the distribution and values of interest.** If the total weight of 30 randomly selected passengers exceeds 6000 pounds, the average weight of these passengers exceeds $\bar{x}$ = 6000/30 = 200 pounds. Because the sample size is large (n = 30 ≥ 30), the distribution of $\bar{x}$ is approximately Normal with $\mu_{\bar{x}} = \mu = 190$ pounds. Because n = 30 is less than 10% of all possible passengers, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{35}{\sqrt{30}} = 6.3901$ pounds. We want to find $P(\bar{x} > 200)$ using the $N(190, 6.3901)$ distribution. **Step 2: Perform calculations. Show your work.** The standardized score for the boundary value is $z = \frac{200-190}{6.3901} = 1.56$. The desired probability is $P(Z > 1.56) = 1 - 0.9406 = 0.0594$. *Using technology*: normalcdf(lower: 200, upper: 1000, μ: 190, σ: 6.3901) = 0.0588. **Step 3: Answer the question.** There is a 0.0588 probability that the mean weight exceeds 200 pounds (and the total weight exceeds 6000 pounds).

7.62 (a) No. A count only takes on whole-number values, so it cannot be normally distributed.
(b) **Step 1: State the distribution and values of interest.** If the total number of people in 700 cars exceeds 1075, the average number of people per car exceeds $\bar{x}$ = 1075/700 = 1.5357 people. Because the sample size is large (n = 700 ≥ 30), the distribution of $\bar{x}$ is approximately Normal with $\mu_{\bar{x}} = \mu = 1.5$ people. Because 700 is less than 10% of all cars entering this interchange, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.75}{\sqrt{700}} = 0.0283$ people. We want to find $P(\bar{x} > 1.5357)$ using the $N(1.5, 0.0283)$ distribution. **Step 2: Perform calculations. Show your work.** The standardized score for the boundary value is $z = \frac{1.5357-1.5}{0.0283} = 1.26$. The desired probability is $P(Z > 1.26) = 1 - 0.8962 = 0.1038$. *Using technology*: normalcdf(lower: 1.5357, upper: 1000, μ: 1.5, σ: 0.0283) = 0.1036. **Step 3: Answer the question.** There is a 0.1036 probability that the mean number of people exceeds 1.5357 (and the total number of people exceeds 1075).

7.63 **Step 1: State the distribution and values of interest.** Because the sample size is large (n = 10,000 ≥ 30), the sampling distribution of $\bar{x}$ is approximately Normal. $\mu_{\bar{x}} = \mu = \$250$. Assuming 10,000 is less than 10% of all homeowners with fire insurance, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1000}{\sqrt{10{,}000}} = \$10$ We want to find $P(\bar{x} \le 275)$ using the $N(250, 10)$ distribution.
Step 2: Perform calculations. Show your work. The standardized score for the boundary value is $z = \frac{275-250}{10} = 2.50$. The desired probability is $P(Z \le 2.50) = 0.9938$. *Using technology*: normalcdf(lower: –1000, upper: 275, μ: 250, σ: 10) = 0.9938. **Step 3: Answer the question.** There is a 0.9938 probability that the mean annual loss from a sample of 10,000 policies is no greater than $275. So the company should be safe basing its rates on the fact that its average loss per policy will be no greater than $275.

7.64 **Step 1: State the distribution and values of interest.** Because the sample size is large (n = 200 $\geq$ 30), the sampling distribution of $\bar{x}$ is approximately Normal with $\mu_{\bar{x}} = \mu = 1.6$. Because 200 is less than 10% of all square yards of this carpet, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1.2}{\sqrt{200}} = 0.085$ We want to find $P(\bar{x} > 1.8)$ using the $N(1.6, 0.085)$ distribution. **Step 2: Perform calculations. Show your work.** The standardized score for the boundary value is $z = \frac{1.8 - 1.6}{0.085} = 2.35$. The desired probability is $P(Z > 2.35) = 1 - 0.9906 = 0.0094$. *Using technology*: normalcdf(lower: 1.8, upper: 1000, μ: 1.6, σ: 0.085) = 0.0093. **Step 3: Answer the question.** There is a 0.0093 probability of finding that the mean number of flaws is greater than 1.8 in a random sample of 200 square yards of carpet.

7.65 b.

7.66 e.

7.67 b.

7.68 d.

7.69 The unemployment rates for each level of education are:

$$P(\text{unemployed} \mid \text{didn't finish HS}) = \frac{(12{,}470 - 11{,}408)}{12{,}470} = \frac{1062}{12{,}470} = 0.0852$$

$$P(\text{unemployed} \mid \text{HS but no college}) = \frac{(37{,}834 - 35{,}857)}{37{,}834} = \frac{1977}{37{,}834} = 0.0523$$

$$P(\text{unemployed} \mid \text{less than bachelor's degree}) = \frac{(34{,}439 - 32{,}977)}{34{,}439} = \frac{1462}{34{,}439} = 0.0425$$

$$P(\text{unemployed} \mid \text{college graduate}) = \frac{(40{,}390 - 39{,}293)}{40{,}390} = \frac{1097}{40{,}390} = 0.0272$$

The unemployment rate decreases with additional education.

7.70 $P(\text{in labor force}) = \frac{12{,}470 + 37{,}834 + 34{,}439 + 40{,}390}{27{,}669 + 59{,}860 + 47{,}556 + 51{,}582} = \frac{125{,}133}{186{,}667} = 0.6704$. There is a 0.6704 probability that a randomly selected person age 25 or older is in the workforce.

7.71 $P(\text{in labor force} \mid \text{college graduate}) = \frac{40{,}390}{51{,}582} = 0.7830$. Given that a randomly selected person is a college graduate, there is a 0.7830 probability that the person is in the labor force.

7.72 The events "in the labor force" and "college graduate" are not independent, because the probability of being in the labor force (0.6704) does not equal the probability of being in the labor force given that the person is a college graduate (0.7830). College graduates are more likely to be in the labor force than other people.